Wiring of No-Signaling Boxes Expands the Hypercontractivity Ribbon

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Joint work with Amin Gohari
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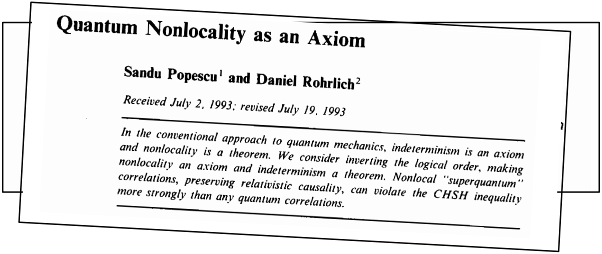
Closed sets of nonlocal correlations

ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL†
Department of Physics, University of Wisconsin, Madison, Wisconsin

(*Received* 4 *November* 1964)

Closed sets of nonlocal correlations



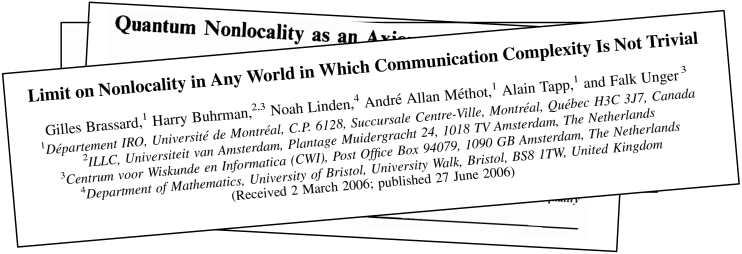
Quantum Nonlocality as an Axiom

Sandu Popescu[1] and Daniel Rohrlich[2]

Received July 2, 1993; revised July 19, 1993

In the conventional approach to quantum mechanics, indeterminism is an axiom and nonlocality is a theorem. We consider inverting the logical order, making nonlocality an axiom and indeterminism a theorem. Nonlocal "superquantum" correlations, preserving relativistic causality, can violate the CHSH inequality more strongly than any quantum correlations.

Closed sets of nonlocal correlations



Quantum Nonlocality as an

Limit on Nonlocality in Any World in Which Communication Complexity Is Not Trivial

Gilles Brassard,[1] Harry Buhrman,[2,3] Noah Linden,[4] André Allan Méthot,[1] Alain Tapp,[1] and Falk Unger[3]
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[2]*ILLC, Universiteit van Amsterdam, Plantage Muidergracht 24, 1018 TV Amsterdam, The Netherlands*
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[4]*Department of Mathematics, University of Bristol, University Walk, Bristol, BS8 1TW, United Kingdom*
(Received 2 March 2006; published 27 June 2006)

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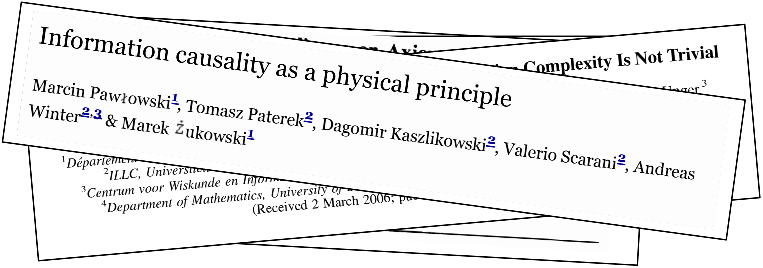

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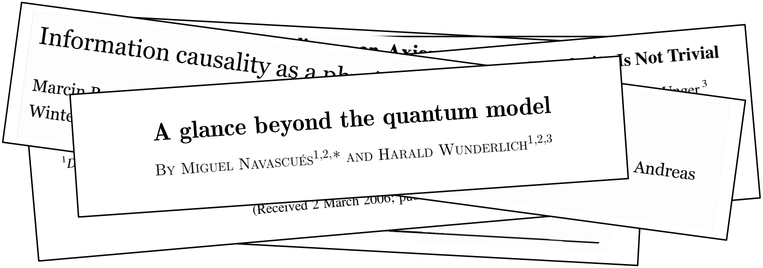

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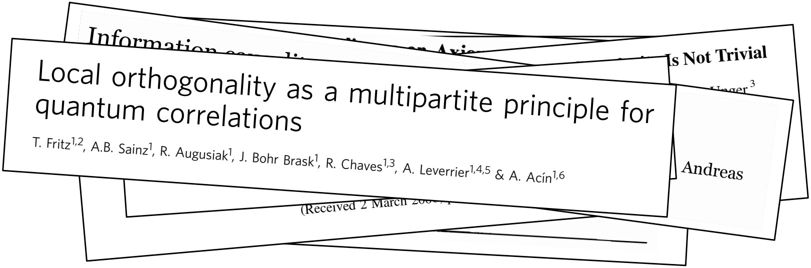

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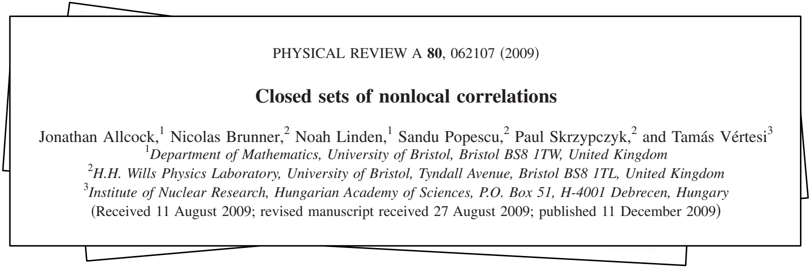

PHYSICAL REVIEW A **80**, 062107 (2009)

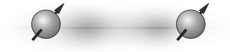
Closed sets of nonlocal correlations

Jonathan Allcock,[1] Nicolas Brunner,[2] Noah Linden,[1] Sandu Popescu,[2] Paul Skrzypczyk,[2] and Tamás Vértesi[3]
[1]*Department of Mathematics, University of Bristol, Bristol BS8 1TW, United Kingdom*
[2]*H.H. Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol BS8 1TL, United Kingdom*
[3]*Institute of Nuclear Research, Hungarian Academy of Sciences, P.O. Box 51, H-4001 Debrecen, Hungary*
(Received 11 August 2009; revised manuscript received 27 August 2009; published 11 December 2009)

Outline

- Introduction to non-local boxes and wirings
- Two measures of correlation with the tensorization property
  - Maximal correlation
  - Hypercontractivity ribbon
- Main result: maximal correlation and hypercontractivity ribbon are monotone under wirings
- Example: simulation of isotropic boxes with each other
  - Resolves a conjecture of Lang, Vértesi, Navascués ’14
- Computability of the above invariants

Local measurements on bipartite physical systems



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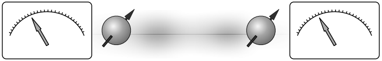

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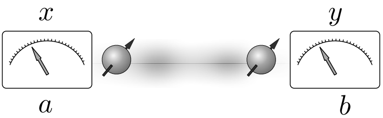

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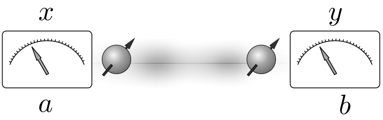

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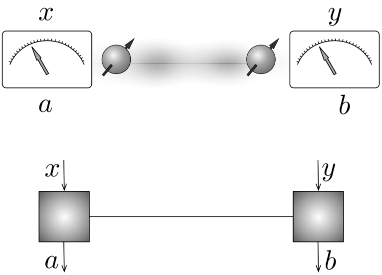

- $p(a, b|x, y) =$ the probability of outcomes a, b under measurement settings x, y

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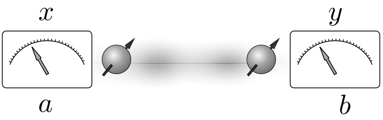
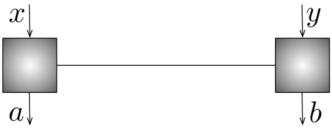


- $p(a,b|x,y) =$ the probability of outcomes a, b under measurement settings x, y

- No-signaling: instantaneous signaling is impossible
  - $p(a|xy)$ is independent of y
  - $p(b|xy)$ is independent of x

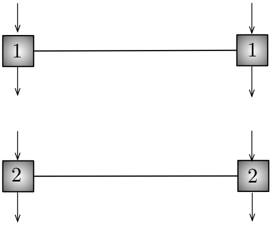
Isotropic boxes



- Example: $x, y, a, b \in \{0, 1\}$, and $0 \leq \eta \leq 1$

$$\mathrm{PR}_\eta(a, b|x, y) := \begin{cases} \frac{1+\eta}{4} & \text{if } a \oplus b = xy, \\ \frac{1-\eta}{4} & \text{otherwise.} \end{cases}$$

Wirings



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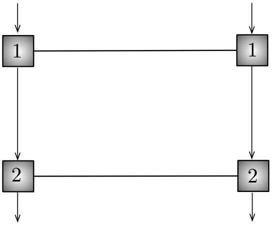

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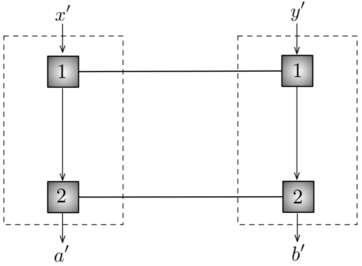

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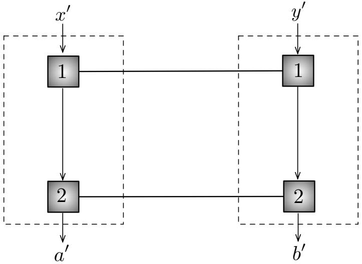

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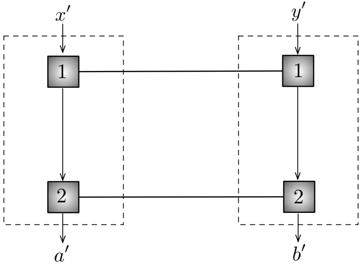

- Wirings are the local operations in the box world
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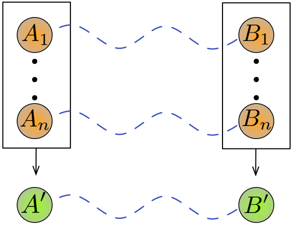
- Problem: $1/2 \leq \eta' < \eta \leq 1$.
  Can we generate PR_η from some copies of $\mathrm{PR}_{\eta'}$ under wirings?
    - No if there are two [Short '09] or at most nine [Forster '11] copies of $\mathrm{PR}_{\eta'}$ available

Tensorization of measures of correlation

- Problem: Given some samples of p_{AB} can we generate one sample from $q_{A'B'}$ under local operations?



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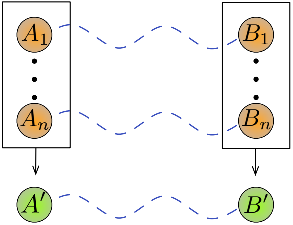
- $I(A,B)_p < I(A',B')_q \;\; \Rightarrow \;\;$ No



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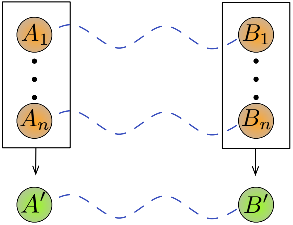

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$$I(A^n,B^n)_{p^n} = nI(A,B)_p.$$

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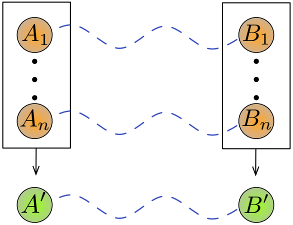

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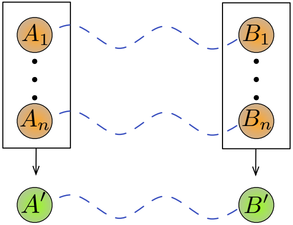

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  - Maximal correlation
  - Hypercontractivity ribbon

Maximal correlation

- Bipartite distribution p_{AB}

$$\rho(A,B) := \max \frac{\mathrm{Cov}(f,g)}{\sqrt{\mathrm{Var}[f_A]\mathrm{Var}[g_B]}} \qquad f_A : \mathcal{A} \to \mathbb{R}, \quad g_B : \mathcal{B} \to \mathbb{R}$$

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- Maximal correlation for non-local boxes:

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Maximal correlation under wirings

Lemma: For any no-signaling box $p(ab|xy)$ we have

$$\rho(A, B) \leq \max\{\rho(A, B|X, Y), \rho(X, Y)\}.$$

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Proof:

$$\begin{aligned}
\mathbb{E}[fg] &= \mathbb{E}_{XY}\mathbb{E}_{AB|XY}[fg] \\
&\leq \mathbb{E}_{XY}\left[\mathbb{E}_{A|XY}[f]\cdot\mathbb{E}_{B|XY}[g] + \rho\sqrt{\mathrm{Var}_{A|XY}[f]\cdot\mathrm{Var}_{B|XY}[g]}\right] \\
&= \mathbb{E}_{XY}\left[\mathbb{E}_{A|X}[f]\cdot\mathbb{E}_{B|Y}[g]\right] + \rho\mathbb{E}_{XY}\left[\sqrt{\mathrm{Var}_{A|X}[f]\cdot\mathrm{Var}_{B|Y}[g]}\right] \\
&\leq \mathbb{E}_X\mathbb{E}_{A|X}[f]\cdot\mathbb{E}_Y\mathbb{E}_{B|Y}[g] + \rho\sqrt{\mathrm{Var}_X\mathbb{E}_{A|X}[f]\cdot\mathrm{Var}_Y\mathbb{E}_{B|Y}[g]} + \rho\mathbb{E}_{XY}\left[\sqrt{\mathrm{Var}_{A|X}[f]\cdot\mathrm{Var}_{B|Y}[g]}\right] \\
&\leq \mathbb{E}_X\mathbb{E}_{A|X}[f]\cdot\mathbb{E}_Y\mathbb{E}_{B|Y}[g] + \rho\sqrt{\mathrm{Var}_X\mathbb{E}_{A|X}[f]\cdot\mathrm{Var}_Y\mathbb{E}_{B|Y}[g]} + \rho\sqrt{\mathbb{E}_X\mathrm{Var}_{A|X}[f]\cdot\mathbb{E}_Y\mathrm{Var}_{B|Y}[g]} \\
&\leq \mathbb{E}_{AX}[f]\cdot\mathbb{E}_{BY}[g] + \rho\sqrt{\left(\mathrm{Var}_X\mathbb{E}_{A|X}[f] + \mathbb{E}_X\mathrm{Var}_{A|X}[f]\right)\left(\mathrm{Var}_Y\mathbb{E}_{B|Y}[g] + \mathbb{E}_Y\mathrm{Var}_{B|Y}[g]\right)} \\
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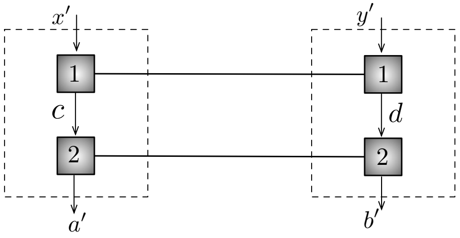
\end{aligned}$$

Maximal correlation under wirings



Theorem

Maximal correlation of no-signaling boxes does not increase under wirings.

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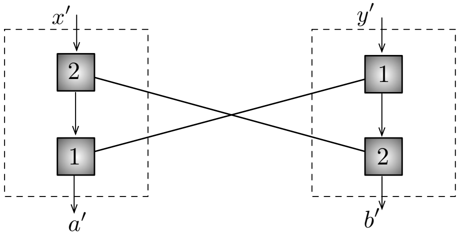

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The proof doesn't work for these types of wirings! We need new tools.

Hypercontractivity ribbon

- [Ahlswede, Gács '76] $(\lambda_1, \lambda_2) \in \mathfrak{R}(A, B)$ iff

$$\mathbb{E}[f_A g_B] \leq \|f_A\|_{\frac{1}{\lambda_1}} \|g_B\|_{\frac{1}{\lambda_2}}, \qquad \forall f_A, g_B$$

Schatten norm: $\|f_A\|_{\frac{1}{\lambda_1}} = \mathbb{E}\left[|f_A|^{1/\lambda_1}\right]^{\lambda_1}$

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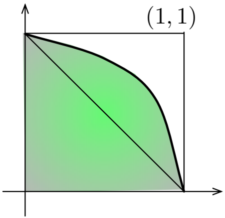

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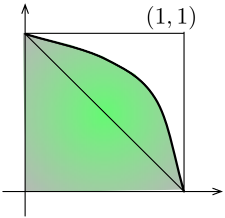

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Hypercontractivity ribbon for non-local boxes:

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Suppose that a no-signaling box $p(a'b'|x'y')$ can be generated from some copies of a box $p(ab|xy)$ under wirings. Then

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Proof: Chain rule!

Example: Isotropic boxes

$$\mathrm{PR}_\eta(a,b|x,y) := \begin{cases} \frac{1+\eta}{4} & \text{if } a \oplus b = xy, \\ \frac{1-\eta}{4} & \text{otherwise.} \end{cases}$$

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- For $0 \leq \eta' < \eta \leq 1$, using an arbitrary number of copies of $\mathrm{PR}_{\eta'}$, a single copy of PR_η cannot be generated under wirings.

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- For $1/\sqrt{2} \leq \eta' < \eta \leq 1$, using an arbitrary number of copies of $\mathrm{PR}_{\eta'}$, a single copy of PR_η cannot be generated under wirings with shared randomness.

Ribbon in terms of a lower convex envelope

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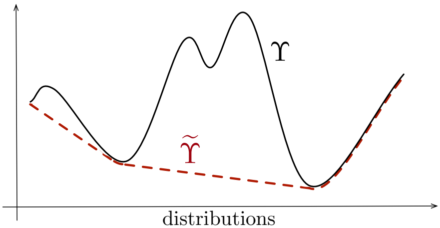

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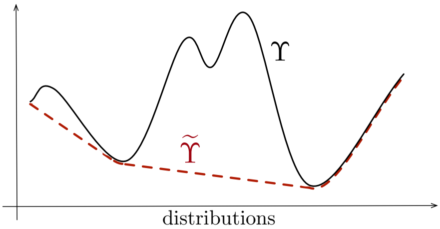
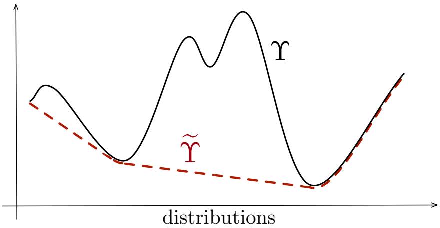


Lemma

For every distribution p_{AB}, we have $(\lambda_1, \lambda_2) \in \mathfrak{R}(A, B)$ if and only if $\Upsilon(p_{AB}) = \widetilde{\Upsilon}(p_{AB})$.

Maximal correlation ribbon



- Definition: $(\lambda_1, \lambda_2) \in \mathfrak{S}(A, B)$ if

$$\mathrm{Var}[f] \geq \lambda_1 \mathrm{Var}_A \mathbb{E}_{B|A}[f] + \lambda_2 \mathrm{Var}_B \mathbb{E}_{A|B}[f], \qquad \forall f_{AB}$$

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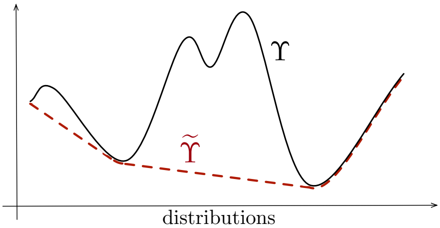

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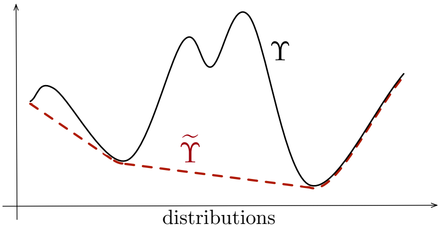

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- $\mathfrak{R}(A, B) \subseteq \mathfrak{S}(A, B)$
- [Tensorization]: $\mathfrak{S}(A^n, B^n) = \mathfrak{S}(A, B)$
- [Data processing]: $\mathfrak{S}(\cdot, \cdot)$ expands under local operations

Maximal correlation ribbon for non-local boxes

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$$\mathfrak{S}(A, B|X, Y) := \bigcap_{x,y} \mathfrak{S}(A, B|X = x, Y = y).$$

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Theorem

Suppose that a no-signaling box $p(a'b'|x'y')$ can be generated from some copies of box $p(ab|xy)$ under wirings. Then

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- Maximal correlation is monotone under wirings

Summary

- Introduced hypercontractivity ribbon for non-local boxes and Showed that it expands under wirings
- Defined Maximal correlation ribbon
- Showed that maximal correlation ribbon expands under wirings
- Characterized maximal correlation in terms of maximal correlation ribbon
- Maximal correlation is monotone under wirings
- There is a continuum of closed sets of boxes
  - Was a conjecture [Lang, Vértesi, Navascués '14]